Sample Question Paper - 7
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank? **[2]**

OR

Find the roots of the quadratic equation : $2x^2 + x + 4 = 0$ by applying the quadratic formula:

2. A boiler is in the form of a cylinder 2 m long with hemispherical ends each of 2 metre diameter. Find the volume of the boiler. **[2]**

3. Given below is the frequency distribution of the heights of players in a school **[2]**

Height(in cm)	160 - 162	163 - 165	166 - 168	169 - 171	172 - 174
Number of students	15	118	142	127	18

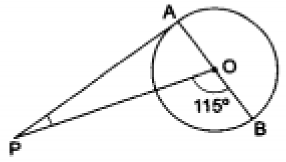
Find the modal height and interpret it.

4. Find the common difference. Given a = first term = -18, n = 10, a_n = the nth term = 0, d = common difference =? **[2]**

5. Find the value of p, if the mean of the following distribution is 7.5. **[2]**

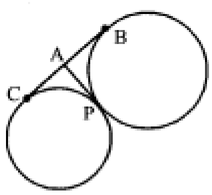
x	3	5	7	9	11	13
f	6	8	15	p	8	4

6. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115°$, find $\angle APO$. **[2]**



OR

In the adjoining figure, BC is a common tangent to the given circles which touch externally at P. Tangent at P meets BC at A. If BA = 2.8 cm, then what is the length of BC?



Section B

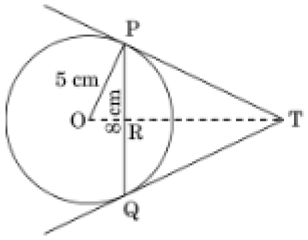
7. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, then find the 21st term of the A.P. **[3]**

8. A tower is 50m high. Its shadow is x metres shorter when the sun's altitude is 45° than when it is 30°. Find the value of x. [Given $\sqrt{3}$ = 1.732.] **[3]**

OR

The angle of elevation of the top of a tower from a point A on the ground is 30^0. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increases to 60^0. Find the height of the tower and the distance of the tower from the point A.

9. In a given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP. **[3]**



10. Solve: $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} [x \neq 0, x \neq -(a+b)]$ **[3]**

Section C

11. Draw a circle of radius 6 cm. Draw a tangent to this circle making an angle of 30° with a line passing through the centre. **[4]**

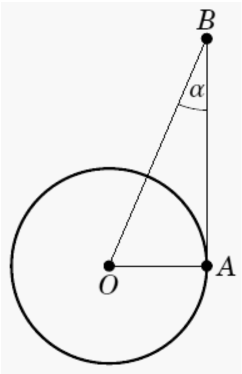
OR

Divide a line segment of length 10 cm internally in the ratio 3: 2.

12. The median of the following data is 16. Find the missing frequencies a and b if the total of frequencies is 70. **[4]**

Class	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40
Frequency	12	a	12	15	b	6	6	4

13. Let O be the center of the earth. Let A be a point on the equator, and let B represent an object (e.g. a star) in space, as shown in the figure. If the earth is positioned in such a way that the angle $\angle OAB = 90^0$, then we say that the angle $\alpha = \angle OBA$ is the equatorial parallax of the object. **[4]**

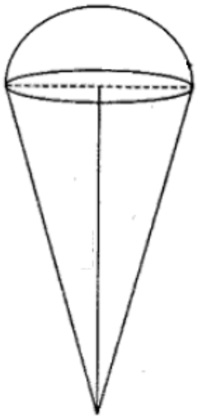


The equatorial parallax of the sun has been observed to be approximately α = 0.00244°. The radius of the earth is 3958.8 miles. Given: sin α = 4.26×10^{-5} and tan α = 4.25×10^{-5}

i. Estimate the distance from the center of the earth to the sun.

ii. Can we say in this problem points O and A are approximately the same points? If yes, how?

14. An 'ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. **[4]**



By reading the above-given information, find the following:

i. The volume of the ice-cream without hemispherical end.

ii. The volume of the ice-cream with a hemispherical end.

Solution

MATHEMATICS BASIC 241

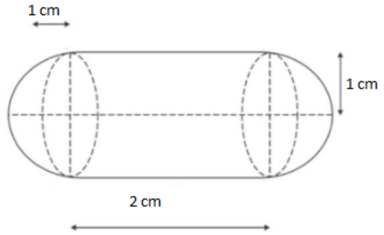
Class 10 - Mathematics

Section A

1. Two tap running together fill the tank in $3\frac{1}{13}$ hr.
   $= \frac{40}{13}$ hours
   If first tap alone fills the tank in x hrs.
   Then second tap alone fills it in (x + 3) hr
   Now $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$
   $\frac{x+3+x}{x(x+3)} = \frac{13}{40}$
   $\frac{2x+3}{x^2+3x} = \frac{13}{40}$
   $80x + 120 = 13x^2 + 39x$
   or, $13x^2 - 41x - 120 = 0$
   $13x^2 - (65 - 24)x + 120 = 0$
   (x - 5)(13x + 24) = 0
   $x = 5, x = -\frac{24}{13}$
   time can't be negative
   Hence, 1st tap takes 5 hours and IInd tap
   takes = 5 + 3 = 8 hours

OR

   We have given that $2x^2 + x + 4 = 0$
   Comparing it with standard form of quadratic equation,
   $ax^2 + bx + c$
   we get, a = 2, b = 1, c = 4
   The roots are given as x $= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
   $= \frac{[-1\pm\sqrt{1-4(2)(4)}]}{2\times2} = \frac{-1\pm\sqrt{1-32}}{4} = \frac{-1\pm\sqrt{-31}}{4}$
   This is not possible, Hence the roots do not exists.

2. 

   According to the question,we are given that,
   Diameter of common base = 2 m
   Then, radius of common base = $\frac{2}{2}$ = 1 m
   Height of the cylinder = 2 m
   Volume of boiler = Volume of cylinder + 2(Volume of hemisphere)
   $= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$
   $= \frac{22}{7} \times 1 \times 1 \times 2 + 2 \times \frac{2}{3} \times \frac{22}{7} \times 1 \times 1 \times 1$
   $= \frac{44}{7} + \frac{88}{21}$
   $= \frac{132+88}{21}$
   $= \frac{220}{21} \text{m}^3$

3. The given data is an inclusive series. So, we convert it into an exclusive form, as given below.

Class	159.5 - 162.5	162.5 - 165.5	165.5 - 168.5	168.5 - 171.5	171.5 - 174.5
Frequency	15	118	142	127	18

Clearly, the modal class is 165.5 - 168.5 as it has the maximum frequency.

$\therefore x_k = 165.5, h = 3, f_k = 142, f_{k-1} = 118, f_{k+1} = 127$

Mode, $M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$

$= 165.5 + \left\{3 \times \frac{(142-118)}{(2\times142-118-127)}\right\}$

$= 165.5 + \left\{\frac{3\times24}{39}\right\}$

$= 165.5 + \frac{24}{13}$

= 165.5 + 1.85

= 167.35 cm

This means that height of maximum number of players in the school is 167.35 cm(approx.).

4. a = a + (n - 1)d

$\Rightarrow 0 = -18 + (10-1)d$

$\Rightarrow 18 = 9d$

$\Rightarrow d = \frac{18}{9} = 2$

5.

x_i	f_i	$f_i x_i$
3	6	18
5	8	40
7	15	105
9	p	9p
11	8	88
13	4	52
	$\sum f_i = 41 + p$	$\sum f_i x_i = 303 + 9p$

$\Sigma f_i = 41 + p, \Sigma f_i x_i = 303 + 9p$

$\therefore \quad \text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 7.5 = \frac{303+9p}{41+p}$

$\Rightarrow 7.5(41 + p) = 303 + 9p$

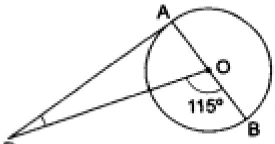
⇒ 307.5 + 7.5p =303 + 9p

⇒9p - 7.5p =307.5 - 303

⇒ 1.5p = 4.5

⇒ p =3

6.



We know that the tangent at a point to a circle is perpendicular to the radius passing through the point of contact.

$\therefore \quad \angle OAP = 90^\circ$

Now, $\angle AOP + \angle BOP$ = 180°

$\Rightarrow \angle AOP = 180^\circ - \angle BOP$

= 180° -115°

= 65°.

Now, $\angle OAP + \angle AOP + \angle APO = 180^\circ$ [sum of angles of a triangle is 180°]

$\Rightarrow \quad \angle APO = 180^\circ - (\angle OAP + \angle AOP)$

= 180° - (90° + 65°) = 25°.

OR

Length of the tangents drawn from an external point to a circle are equal.

$\therefore CA = BA = 2.8cm$ ...(i)

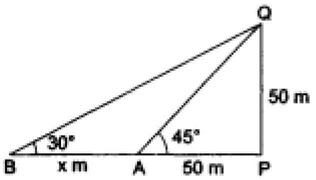
$AB = AP = 2.8cm$ ...(ii)
From equation (i) and (ii) :
$CA = AB = 2.8cm$
$CB = CA = AB$
$\therefore BC = 2.8 + 2.8$
BC = 5.6 cm

Section B

7. Given, a = 10, and S_{14} = 1050

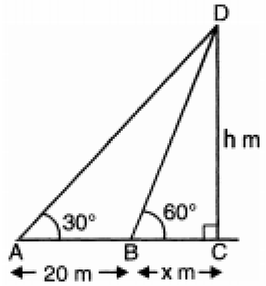
Let the common difference of the A.P. be d
we know that $S_n = \frac{n}{2}[2a + (n-1)d]$
$\therefore S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$
1050 = 7(20 + 13d)
or 20 + 13d = $\frac{1050}{7}$
20 + 13d = 150
13d = 150 - 20
13d = 130
d = $\frac{130}{13}$
d = 10
Now, a_{21} = a + (n − 1)d
= 10 + (21 - 1) 10
= 10 + 20 × 10
= 10 + 190
= 210
Hence, a_{20} = 210

8. Let PQ be the tower and let PA and PB be its shadows when the altitudes of the sun are 45° and 30° respectively. Then,



$\angle PAQ = 45^\circ, \angle PBQ = 30^\circ, \angle BPQ = 90^\circ, PQ = 50\text{m}.$
Let $AB = x\ m.$
From right ΔAPQ, we have
$\frac{AP}{PQ} = \cot 45^\circ = 1$
$\Rightarrow \frac{AP}{50\text{m}} = 1 \Rightarrow AP = 50\text{m}.$
From right ΔBPQ, we have
$\frac{BP}{PQ} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{x+50}{50} = \sqrt{3} \Rightarrow \quad x = 50(\sqrt{3}-1).$
$\Rightarrow x = 50(1.732 - 1) = (50 \times 0.732) = 36.6$
Hence, $x = 36.6$

OR



Let height of tower be h m and distance BC be x m
In $\triangle$ DBC, $\frac{h}{x} = \tan 60^\circ$

$\Rightarrow \quad h=\sqrt{3x}$ .....(i)

$\frac{h}{x+20}=\tan 30^{\circ}=\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3h}=x+20$ ....(ii)

Substituting the value of h from eq. (i) in eq. (ii), we get

3x = x + 20

3x - x = 20

Or 2x = 20

$\Rightarrow$ x = 10 m ...(iii)

Again$h=\sqrt{3x}$

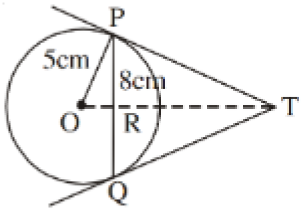
or, $h=\sqrt{3}\times 10=10\sqrt{3}$

$=10\times 1\cdot 732$

= 17.32 m

[from (i) and (in)]

Hence, height of tower is 17.32 m and distance of tower from point A is 30 m

9. 

Let TR be x cm and TP be y cm

OT is perpendicular bisector of PQ

So PR = 4 cm (PR = $\frac{PQ}{2}=\frac{8}{2}$)

In $\triangle$OPR, $OP^2 = PR^2 + OR^2$

$5^2 = 4^2 + OR^2$

OR = $\sqrt{25-16}$

$\therefore$ OR = 3 cm

In $\triangle$PRT, $PR^2 + RT^2 = PT^2$

$y^2 = x^2 + 4^2$ .....(1)

In $\triangle$OPT, $OP^2 + PT^2 = OT^2$

$(x + 3)^2 = 5^2 + y^2$ (OT = OR + RT = 3 + x)

$\therefore$ $(x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving, we get x = $\frac{16}{3}$ cm

From (1), $y^2 = \frac{256}{9} + 16 = \frac{400}{9}$

So, y = $\frac{20}{3}$cm = 6.667 cm

10. Given,

$\frac{1}{(a+b+x)}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

$\Rightarrow \quad \frac{1}{(a+b+x)}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}\Rightarrow \frac{x-(a+b+x)}{x(a+b+x)}=\frac{b+a}{ab}$

$\Rightarrow \quad \frac{-(a+b)}{x(a+b+x)}=\frac{(a+b)}{ab}$

On dividing both sides by (a+b)

$\Rightarrow \quad \frac{-1}{x(a+b+x)}=\frac{1}{ab}$

Now cross multiply

$\Rightarrow$ x(a + b + x) = -ab

$\Rightarrow$ x^2 + ax + bx + ab = 0

$\Rightarrow$ x(x +a) + b(x +a) = 0

$\Rightarrow$ (x + a) (x + b) = 0

$\Rightarrow$ x + a = 0 or x + b = 0

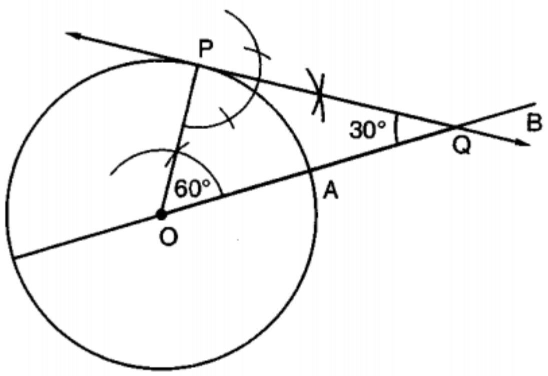
$\Rightarrow$ x = -a or x = -b.

Therefore, -a and -b are the roots of the equation.

Section C

11. Steps of construction

STEP I Draw a circle with centre O and radius 3 cm.



STEP II Draw a radius OA of this circle and produce it to B.

STEP III Construct an angle $\angle AOP$equal to the complement of 30° i.e. equal to 60°.

STEP IV Draw perpendicular to OP at P which intersects OA produced at Q

Clearly, PQ is the desired tangent such at $\angle OQP$ = 30°

OR

We follow the following steps of construction.

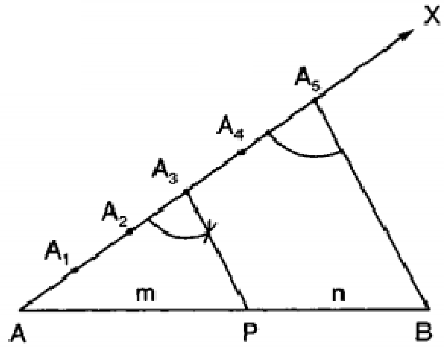


Fig. 9.2

Steps of construction

STEP I Draw a line segment AB = 10 cm by using a ruler.

STEP II Draw a ray AX making an acute angle$\angle BAX$ with AB.

STEP III Along AX, mark-off 5 (= 3 + 2) points A_1, A_2, A_3, A_4 and A_5 such that

$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.......(1)

STEP IV Join points B & A_5 .

STEP V Through A_3 draw a line A_3P parallel to A_5 B by drawing angle AA_3P equals to angle AA_5B. A_3P intersects AB at point P. Since, $AA_3 : A_3A_5$ = 3:2 [from(1) & figure] . Thus, AP : PB = 3:2. (due to symmetry)

Hence, point P divides AB internally in 3:2.

12. Let the missing frequencies are a and b.

Class Interval	Frequency f_i	Cumulative frequency
0 - 5	12	12
5 - 10	a	12 + a
10 - 15	12	24 + a
15 - 20	15	39 + a
20 - 25	b	39 + a + b
25 - 30	6	45 + a + b
30 - 35	6	51 + a + b

35 - 40	4	55 + a + b = 70

Then, 55 + a + b = 70

a + b = 15 ......(1)

Median is 16, which lies in 15 - 20

So, The median class is 15 - 20

Therefore, l = 15, h = 5, N = 70, f = 15 and cf = 24 + a

Median is 16, which lies in the class 15 - 20. Hence, median class is 15 - 20.

$\therefore l = 15, h = 5, f = 15, c.f. = 24 + a$

Now, Median = $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$\therefore 16 = 15 + \left\{5 \times \frac{(35-24-a)}{15}\right\}$

$\Rightarrow 16 = 15 + \left\{\frac{11-a}{3}\right\}$

$\Rightarrow 1 = \frac{11-a}{3}$

$\Rightarrow 3 = 11 - a$

$\Rightarrow a = 8$

Now, $55 + a + b = 70$

$\Rightarrow 55 + 8 + b = 70$

$\Rightarrow 63 + b = 70$

$\Rightarrow b = 7$

Hence, the missing frequencies are a = 8 and b = 7.

13. i. Given: α = 0.00244^{0}

OA = Radius of the earth = 3958.8 miles

Since $\angle$OAB = 90^{0}, we have

sin α = $\frac{OA}{OB}$

OB = $\frac{OA}{\sin\alpha}$ = $\frac{3958.8}{\sin 0.00244}$ = 92960054.1 = 93 million (approx)

So, the distance from the center of the earth to the sun is approximately 93 million miles.

ii. Now, tan α = $\frac{OA}{AB}$

AB = $\frac{OA}{\tan\alpha}$ = $\frac{3958.8}{\tan 0.00244}$ = 92960054.02 = 93 million (approx)

As, OB and AB are approx equal, so we can say points O and A are approximately the same points in this problem.

14. For cone, Radius of the base (r)

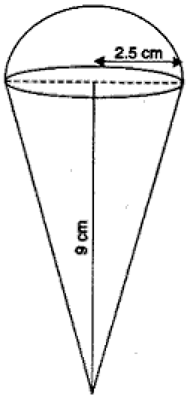
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. The volume of the ice-cream without hemispherical end = Volume of the cone

$= \frac{825}{14}\text{cm}^3$

ii. Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$

$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$